Introduction and Intermediate Algebra

7.1: Simplifying Rational Expressions

Rational Expression

A fraction whose numerator and denominator are polynomials is called a **rational expression.**

$$\frac{3}{x + 4} \qquad \frac{2x}{x^2 - 4x - 4} \qquad \frac{x^2 - 5x}{x^2 + 2x - 3}$$

You have learned that because division by zero is undefined, the denominator of a rational expression cannot be zero.

Rational Expression

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$$\frac{3}{x+4} \qquad \frac{\cancel{2x}}{x^2+x-6} \qquad \frac{\cancel{x^2-5x}}{x^2+2x-3}$$

$x+4=0$

$-4 \quad -4$

$x=-4$

$x\neq-4$

$(x-2)(x+3)=0$

$x-2=0 \qquad x+3=0$

$+2 \quad +2 \qquad -3 \quad -3$

$x\neq2 \qquad x\neq-3$

$(x+3)(x-1)$

$x+3=0 \qquad x-1=0$

$x\neq-3 \qquad x\neq1$

What can x not equal in each expression?
(What would make the expression undefined?)

The set of *usable* values of the variable is called the domain of the rational expression.

What is the domain of these expressions?

The set of *usable* values of the variable is called the domain of the rational expression.

$$\frac{3}{x+4}$$

$x \neq -4$

The domain is everything except -4.

-4

$(-\infty, -4) \cup (-4, \infty)$

$\{x \mid x \neq -4\}$

$$\frac{2x}{x^2+x-6}$$

$x \neq -2, 3$

The domain is everything except -2 and 3.

$-2 \qquad 3$

$(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

$\{x \mid x \neq -2, 3\}$

$$\frac{x^2-5x}{x^2+2x-3}$$

$x \neq -1, 3$

The domain is everything except -1 and 3.

$-1 \qquad 3$

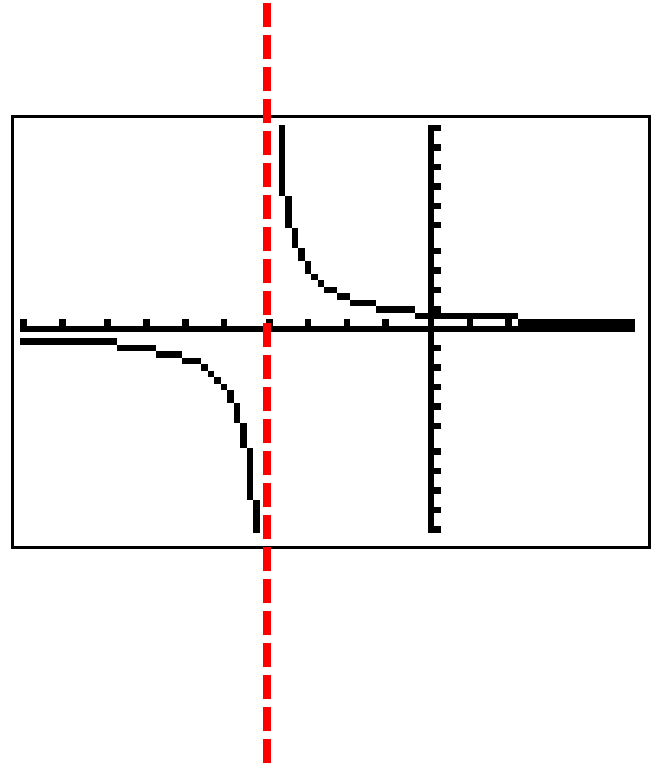
$(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$

$\{x \mid x \neq -1, 3\}$

What does the graph look like?

Just an FYI for College Algebra

$$y = \frac{3}{x+4}$$



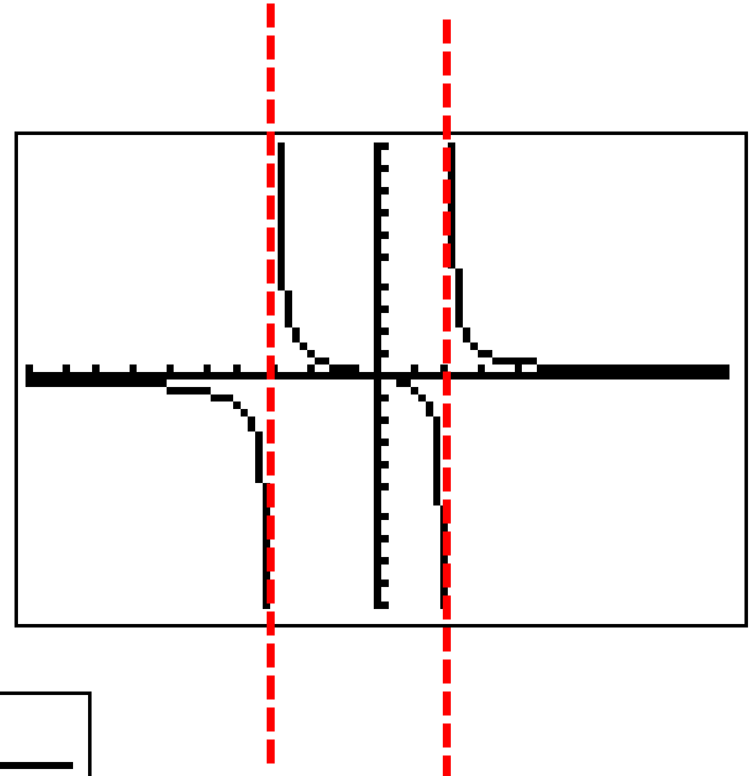
X	Y_1	
-7	-1	
-6	-1.5	
-5	-3	
-4	ERROR	
-3	3	
-2	1.5	
-1	1	

X= -7

What does the graph look like?

Just an FYI for College Algebra

$$y = \frac{2x}{x^2 + x - 6}$$



X	Y_1
-3	ERROR
-2	1
-1	.33333
0	0
1	-.5
2	ERROR
3	1

X= -3

Simplifying Rational Expressions

Let u, v, and w represent real numbers, variables, or algebraic expressions such that $v \neq 0$ and $w \neq 0$. Then the following is valid.

$$\boxed{\frac{uw}{vw} = \frac{u\cancel{w}}{v\cancel{w}} = \frac{u}{v}}$$

Like reducing fractions!

$$\frac{4x}{2(x+1)}$$

You *can* divide out the common factor 2.

$$\frac{\cancel{2}\cdot 2x}{\cancel{2}(x+1)} = \frac{2x}{x+1}$$

$$\frac{2x}{x+1}$$

You *cannot* divide out the common term x.

Simplifying Rational Expressions

Simplifying a rational expression requires two steps:

(1) completely factor the numerator and denominator

(2) divide out any *factors* that are common to both the numerator and denominator.

So, your success in simplifying rational expressions actually lies in your ability to *factor completely* the polynomials in both the numerator and denominator.

Simplify the rational expression:

$$\frac{2x^3-6x}{6x^2} = \frac{\cancel{2x}(x^2-3)}{\cancel{2x}\cdot 3x}$$

completely factor the numerator and denominator

$$\boxed{\frac{x^2-3}{3x}}$$

divide out any *factors* that are common to both the numerator and denominator

0

$6x^2=0$

$x \neq 0$

State the domain for the rational function.

$(-\infty,0)\cup(0,\infty)$

$\{x \mid x \neq 0\}$

Simplify the rational expression:

$$\frac{2x^2 - 4x}{(x-2)^2} = \frac{2x\cancel{(x-2)}}{(x-2)\cancel{(x-2)}}$$

$$\boxed{\frac{2x}{x-2}}$$

completely factor the numerator and denominator

divide out any *factors* that are common to both the numerator and denominator

$\{x \mid x \neq 2\}$

$x \neq 2$

State the domain for the rational function.

$(-\infty, 2) \cup (2, \infty)$

Simplify the rational expression:

$$\frac{x^2 - 4x + 3}{x^2 - 5x + 6}$$

$$\frac{\cancel{(x-3)}(x-1)}{\cancel{(x-3)}(x+2)}$$

$$\frac{x-1}{x+2}$$

3, $-1 + -3 = -4$

6, $-2 + -3 = -5$

completely factor the numerator and denominator

$(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

$\{x \mid x \neq 3, -2\}$

divide out any *factors* that are common to both the numerator and denominator

original

$(x-3)(x+2)$

$x \neq 3 \quad x \neq -2$

State the domain for the rational function.

Simplify the rational expression:

$$\frac{3x^2 - 9x - 12}{6x^2 + 30x + 24} = \frac{3(x^2-3x-4)}{6(x^2+5x+4)}$$

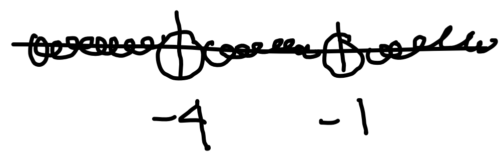
$$\frac{\cancel{3}(x-4)\cancel{(x+1)}}{{}_{2}\cancel{6}(x+4)\cancel{(x+1)}} = \frac{(x-4)}{2(x+4)}$$

completely factor the numerator and denominator

divide out any *factors* that are common to both the numerator and denominator

denom

$x \neq -4 \quad x \neq -1$



$(-\infty,-4)\cup(-4,-1)\cup(-1,\infty)$

State the domain for the rational function.

$\{x \mid x \neq -4, -1\}$

Function Notation

Evaluating functional notation.
$y = f(x)$ is read as
"y is a function of x"

$f(x) = x + 1$

$f(2) = 2 + 1 = 3$ $(2, 3)$

f(2) means to evaluate $x + 1$ when $x = 2$

Let $f(x) = 3x^2 + 2x - 7$

Find $f(4) = 3(4)^2 + 2(4) - 7 = 49$

Find $f(-2) =$

Find $f(a) = 3a^2 + 2a - 7$

Let $f(x) = 3x^2 + 2x - 7$

Find $f(x + 1) =$

$3(x+1)^2 + 2(x+1) - 7$

$3(x+1)(x+1) + 2(x+1) - 7$

$3(x^2+2x+2) + 2x + 2 - 7$

$3x^2 + 6x + 6 + 2x + 2 - 7$

$\boxed{3x^2 + 8x + 1}$

Homework 7.1